

$$\tilde{t} = t + z/c$$

Dirac
 Bjorken, Drell, Srednicki
 Le Bellac + SSB
 Pauli + SSB

Equation of motion

$$i \frac{\partial}{\partial t} |\Psi_n\rangle = P^- |\Psi_n\rangle = \frac{M_n^2 + P_{\perp}^2}{P^+} |\Psi_n\rangle$$

$$H_{LC} = P^- P^+ - P_{\perp}^2$$

$$P^+, P_{\perp}$$

kinematical

$$H_{LC} |\Psi_n\rangle = M_n^2 |\Psi_n\rangle$$

→ Eigenvalue problem for LC Hamiltonian

Insert complete set $\sum_n |n\rangle \langle n| = I$ $\xrightarrow{H_{LC}^0 \text{ eigenstate}}$ color-singlet

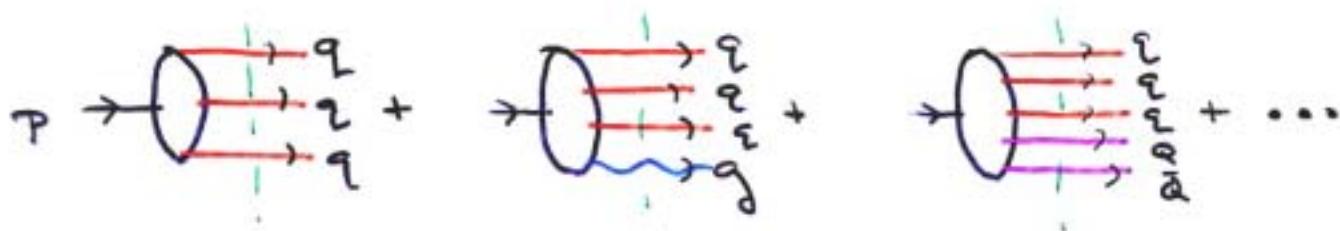
$$\sum_n \langle m | H_{LC} | n \rangle \langle n | \Psi_n \rangle = M_n^2 \langle m | \Psi_n \rangle$$

⇒ Heisenberg matrix form of eigenvalue problem ^{DLCQ}

$$|\Psi_n\rangle = \sum_n |n\rangle \langle n | \Psi_n \rangle = \sum_n |n\rangle \Psi_{n/H}^{(x_i, k_{\mu}, \lambda)}$$

⇒ LC Fock expansion of eigenstate $|\Psi_n\rangle$

Light-Cone Fock Representation of Hadrons



$$|P\rangle = \sum_n |n\rangle \Psi_n(x_i, k_{xi}, \lambda_i)$$

$\wedge \sum_i x_i = 1, \sum_i k_{xi} = 0$

* Explicit solutions using "DLCQ" QCD(1+1), "collinear QCD"
SIB, Pauli, Horubetsky, Antonuccio, Dalley

* Calculate structure functions modulo FSI

$$q(x) \rightarrow g(x), Q(x)$$

spin-dependence

* Calculate Regge behavior using "ladder relations" $x \rightarrow 0$, BFKL

Mueller, SIB, Antonuccio, Dalley

* $x \rightarrow \pm$ constraints Lepage, SIB, Burdman, Schubert

* Properties of heavy quark sea $s(x) \neq \bar{s}(x)$

Koger, Ma, Schmid

extrinsic vs intrinsic
physics of $\Delta \Sigma$, anomaly
Ball

SIB, Schlueter

QCD (3+1)

Sector	Class	0	g	q̄q	gg	q̄g g	gg g	q̄g q̄g	ḡg gg	gg gg	q̄g q̄g g	q̄g gg g	gg gg g
1	0	0											
2	g												
3	g												
4	g												
5	g												
6	g												

1-6075-30-NP-1

DLCQ: $\langle n | H_{\text{FC}}^H | m \rangle$

$$(m^2 - \sum_x \frac{k_x^2 + m^2}{x}) \psi_n = \sum_m \langle n | H_{\text{FC}}^H | m \rangle \psi_m$$

FC
FC

PBC: $k^+ = \frac{2\pi}{L} n$, $k_T = \frac{2\pi}{L} \tilde{n}_T$

DLCQ : Discretized Light-Cone Quantization

H.C. Pauli
SvB

Assume PBC for $-L < x^- < L$

$$P^+ = \frac{2\pi}{L} k_+$$

$$k^\pm = k^0 \pm k^z$$

$$k_+^+ = \frac{2\pi}{L} n_i$$

$$\sum_i n_i = k_+$$

↑
positive integers

∴ Finite Fock Basis ! $\sum_i k_+^2 + n_i^2 < \Delta^2$

DLCQ: Diagonalize first Hermitian Matrix
 $\langle n | H_{LF} | m \rangle$

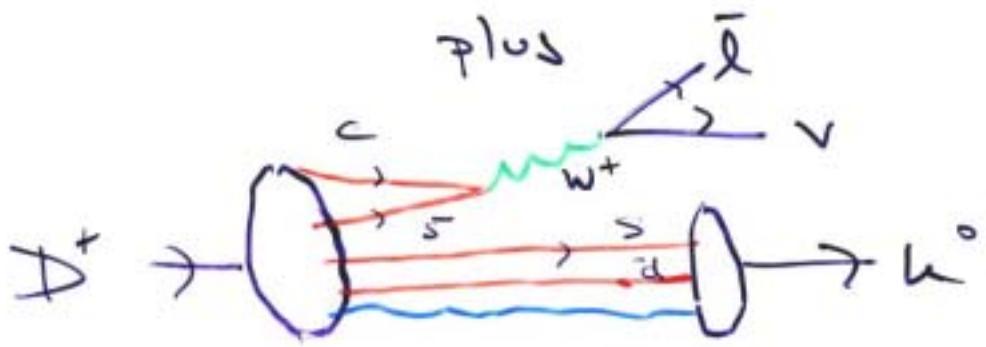
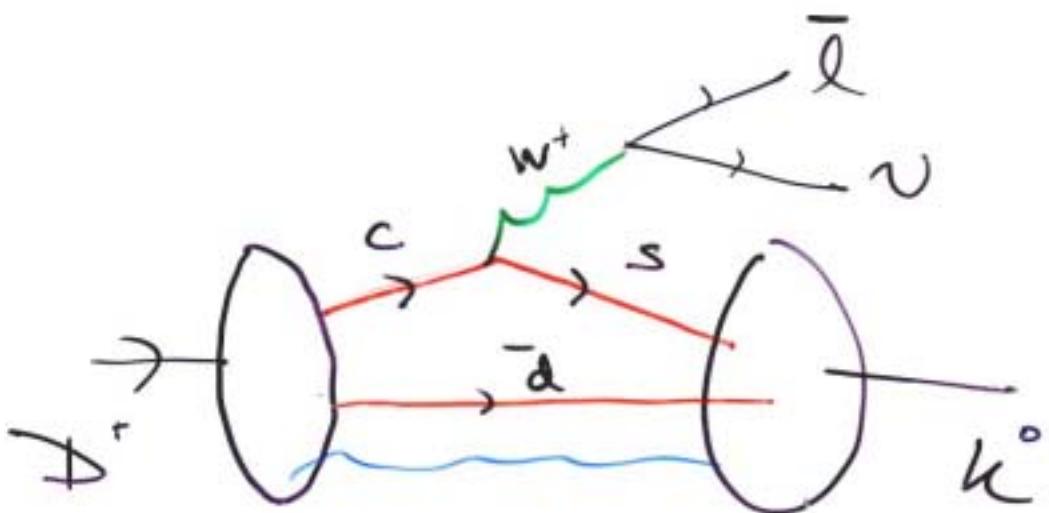
→ Spectrum, LFWFs

Continuum limit $k_+ \rightarrow \infty$

Light-Cone Wavefunctions:

B, D

Exclusive decays



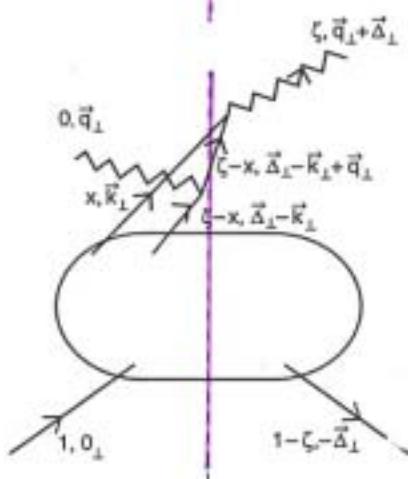
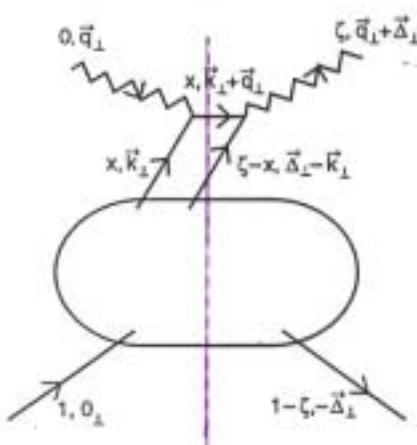
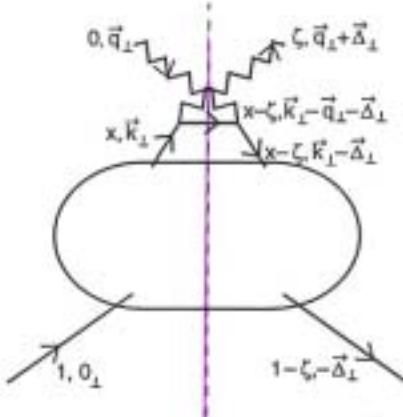
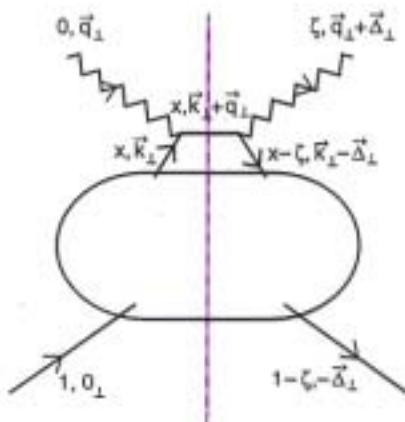
plus
New phenomena
in
weak decays

Ti, Choi

M. Diehl
T. Feldmann
R. Jakob
P. Kroll

M. Diehl
D-S Hwang
+
SJL

$\alpha=0$ frame



5-2000
B590A8

Light-cone time-ordered contributions to deeply virtual Compton scattering.
Only the contributions of leading twist in $1/q^2$ are illustrated. These contributions illustrate the factorization property of the leading twist amplitude.

$\Psi_n \Psi_n$

$\Psi_{n+1} \Psi_{n+1}$

Pauli Form Factor measures proton orbital angular momentum!

S.Drell
SJS

$$F_1 = \Psi'_{Lz} \otimes \Psi_{Lz} \quad \Delta L_z = 0$$

$$\frac{Q}{2M} F_2 = \Psi'_{Lz'} \otimes \Psi_{Lz} \quad \Delta L_z = \pm 1$$

$$F_2, \Sigma \sim \langle \uparrow | \vec{S} \cdot \vec{\Sigma} | \downarrow \rangle$$

Angular Momentum on LF:

(const. rest)
(free)

$$\vec{J} = \vec{n} - i(\vec{h}_z \times \frac{\partial}{\partial \vec{h}_z}) - i(\vec{n} \times \frac{\partial}{\partial \vec{n}})$$

Kennard
Sommer

Explicitly covariant formulation: $\vec{h}^t = \vec{n} \cdot \vec{h}$, $\vec{n} = 0$
general \vec{n}^n

$$\text{Find } \Psi_{Lz=1} \sim m \frac{\vec{S} \cdot \vec{n} \times \vec{h}}{\vec{h}^2} \Psi_{Lz}$$

Hilfer
Huang
Kennard
SJS

- * Confirmed by explicit B.S. basis projected to LF
- * PQCD Belitsky, Ji, Yuan
- * Conformal Symmetry Brown, Fried
- * AdS/CFT Polchinski, Strominger / de Teramond SJS

$$\frac{F_2(Q^2)}{F_1(Q^2)} \rightarrow \frac{1}{Q^2} \text{ modulo } \log Q^2, \text{ not } \frac{1}{Q}$$

Explicitly covariant light-front formalism

Kazakov, Shirman, Carbonell
Hiller, Hwang, Kazakov, S&B

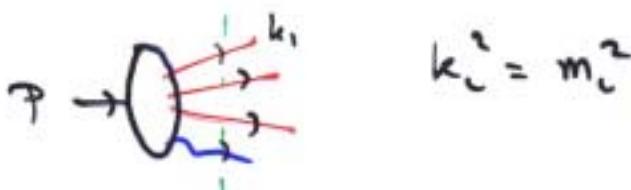
* n^μ can have arbitrary direction $n^i = 0$

$$n^\mu = (n^+, n^-, 0_\perp) \Rightarrow (0, z, 0_\perp) \text{ standard frame}$$

Lorentz scalar LFWF:

$$\phi_{LF}^{j=0} = \phi_{LF}(x_i, m^2)$$

$$x_i = \frac{k_i \cdot n}{p \cdot n}, \quad m^2 = (\sum k_i^2)^2 = \sum_i \left[\frac{k_i^2 + k_i^2}{x_i} \right].$$



examples

$$d=\frac{1}{2} \quad p \rightarrow \begin{array}{c} k \\ \diagdown \\ \text{---} \\ p-k \end{array} \quad \Psi_{LF}^{j=\frac{1}{2}} = \bar{u}(k) [\phi_1 + \frac{m \times}{n \cdot p} \phi_2] u(p)$$

$$d=0 \quad p \rightarrow \begin{array}{c} k \\ \diagdown \\ \text{---} \\ p-k \end{array} \quad \Psi_{LF}^{j=0} = \epsilon \cdot k \phi_1 + \frac{\epsilon \cdot n}{n \cdot p} \phi_2$$

$$\phi_i = \phi_i(x_i, m^2)$$

"Light-Front Invariance"

$$\tau = x \cdot n , \quad A \cdot n = 0 , \quad n^2 = 0$$

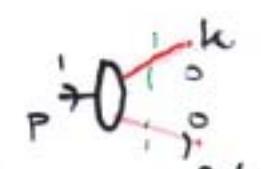
Physical observables must be independent of n^μ

$$\left[n^\mu \times \frac{\partial}{\partial n^\mu} \right] \mathcal{O} = 0$$

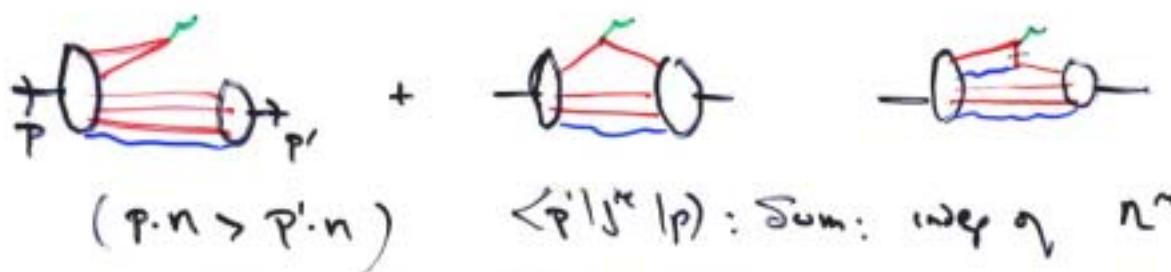
\mathcal{O} = Lorentz invariants : $F_i(q^2)$, $W_{\mu\nu}$, σ
 ψ_{BS} , $\langle p' | j^\mu | p \rangle$
 renorm. constants

L.F. W.F.s depend on n^μ (must allow for all)
 Lorentz const.

Example :

$$(n=1) \quad \psi_{LF}^{J=1} = \epsilon \cdot k \phi_1 \left(\frac{k \cdot n}{p \cdot n}, m \right) + \frac{\epsilon \cdot n}{p \cdot n} \phi_2 \left(\frac{k \cdot n}{p \cdot n}, m \right)$$


($\epsilon \cdot p = 0$)



$$(\text{p} \cdot n > \text{p}' \cdot n) \quad \langle p' | j^\mu | p \rangle : \text{Sum: independent of } n^\mu$$

Light-Front Hadron Dynamics

and AdS/CFT Correspondence

G de Teranow

SBS

$$\text{AdS}_5 \times X_5 \leftrightarrow SO(4,2)$$

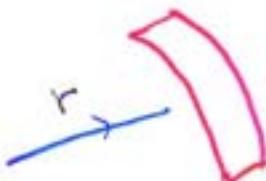
AdS/CFT — Dual to $SO(4,2)$
conformal space-time

$$ds^2 = \frac{r^2}{R^2} g_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 dS_X^2$$

$$R^2 = (4\pi g_s N_c)^{1/2} \alpha' \quad \text{Maldacena}$$

→ $\frac{r^2}{R^2} x^2 \sim \frac{r^2}{r^2} r^2 , \quad x^2 \sim \frac{R^4}{r^2}$

* $x^2 = O(\frac{1}{Q^2}) \Rightarrow r^2 = O(Q^2) \cdot R^4$



large r : conformal domain
 $r \sim r_0$: non-conformal

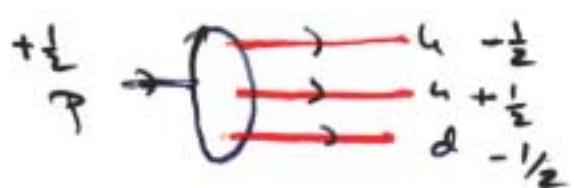
$$\Xi = \Psi(r, x) Y(\Omega_x) , \quad \Psi(r, x) \sim r^{-\Delta} e^{i p \cdot x}$$

large r

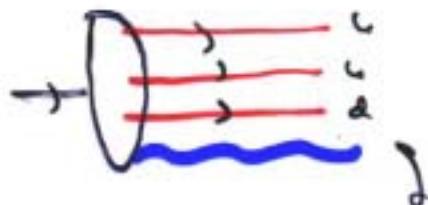
$AdS/CFT \Rightarrow$

* Orbital Angular Momentum

* Higher Fock States



$$L_z = 1$$



$$\psi_{ggg}^{L_z=1} \sim \frac{f_B \left(\frac{1}{\alpha'_{QCD}} \right)^{1/2} \vec{s} \cdot \vec{n} \times \vec{h}_z}{[h_z^2 + \frac{1}{\alpha'_{QCD}}]^{3/2}}$$

$$\psi_{gggg}^{L_z=0} \sim \frac{f_B \left(\frac{1}{\alpha'_{QCD}} \right)^{1/2} \vec{s} \cdot \vec{n} \times \vec{\epsilon}_z}{[h_z^2 + \frac{1}{\alpha'_{QCD}}]^{3/2}}$$

$$\psi_{ggg\gamma}^{L_z=1} \sim \frac{f_B \left(\frac{1}{\alpha'_{QCD}} \right)^{1/2} \vec{h}_z (\vec{n} \times \vec{\epsilon}_z)}{[h_z^2 + \frac{1}{\alpha'_{QCD}}]^{3/2}}$$

$$\Rightarrow F_2(Q^2) = \frac{1}{a} \int \psi_{ggg}^{L_z=0} \psi_{ggg}^{L_z=1} d^2 h_z d\epsilon \sim \frac{1}{Q^6} \frac{1}{\alpha'^{1/2}_{QCD}}$$

$\times \log s$

Consequences \ Conformal Scaling of $\Psi_{n/h}(x, k_z, \lambda)$

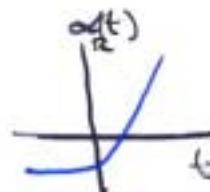
* $F_{\alpha S_2=0}(q^2) \sim \frac{1}{(q^2)^{n-1}}$ mod loss/
anom dim.

* $q^2 F_2(q^2) / F_1(q^2) \sim \text{const}$ mod loss/
anom dim.

* No large suppression of spin-1 contrib.

* Dominance of quark interchange

* $\alpha_R(t) \rightarrow \text{neg. integer}$



* $\frac{d\sigma}{dt}(AB \rightarrow CD) \sim \frac{1}{g_{\text{min}}^2} F(t_B)$

* Non pert. Normalization: $\sum C_n \alpha_s^n \Rightarrow \sqrt{\alpha_s}$

* Orbital dependence of LFWF

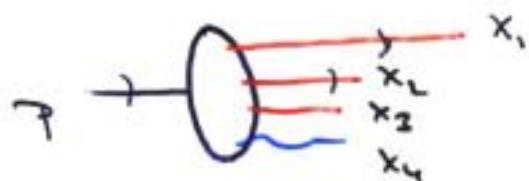
Dimitrov, Kormann
Hiller, Hwang, Kormann
SBS

* x-ray of distribution amplitudes

D. Müller

V. Basso, P. Ball
Korchovsky

Structure Functions at $x \rightarrow 1$



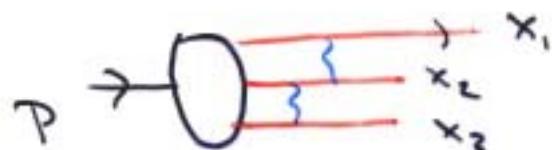
$$M_n^2 = \sum_{i=1}^n \frac{b_i^2 + m^2}{x_i}$$

$$\rightarrow \infty \quad \begin{cases} x_i \rightarrow 1 \\ x_{i \neq 1} \rightarrow 0 \end{cases}$$

$$\sum x_i = 1$$

SUB, Gunion, Soper
Frixon + Soper
SOP + CPL

Iterate kernel:



Boulat, Schmidt, SOLO
Leading order

$$q(x) \sim (1-x)^{2n_5 - 1 + 2\Delta h} \quad u/d = 5:1$$

$$= \begin{cases} (1-x)^3 & q \uparrow \\ (1-x)^5 & q \downarrow \end{cases}$$

$$q(x) \sim \begin{cases} (1-x)^4 & g \uparrow \\ (1-x)^6 & g \downarrow \end{cases}$$

note: minimal evolution at $x \rightarrow 1$ (off shell effect)

Compare String Theory / Dim. Counting
with PQCD Scaling

$$2 \rightarrow 2 \quad \frac{1}{N^2} \rightarrow \frac{1}{N} \quad \text{for quark composites}$$

* Higher order corrections in α_s ?

Exact in string theory!

* Anom. Dimension from Evolution of $\phi_h(x, Q)$

$$e^{Y_u g(Q^2, Q_0^2)} = \begin{cases} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{\gamma_u C_F/\beta} & \text{NSF} \\ \left(\frac{Q^2}{\Lambda^2}\right)^{\frac{\alpha_s C_F}{4\pi} Y_u} & \text{conf} \end{cases}$$

$$g(Q^2, Q_0^2) = \frac{C_F}{4\pi} \int_{Q_0^2}^{Q^2} \frac{d\lambda^2}{\lambda^2} d_s(\lambda^2)$$

* Pinch contributions: suppressed by $(Q^2)^{-C_F \ln Q^2}$
in conf. theory

large $N_C \Rightarrow$ suppressed

Ferm + BGD
Malden Theory

* What is magnitude of α_s in conf. theory?

Analytic properties of Form factors

$$Q_i = Q_i(x_i, m^2)$$

Compute Form factors from overlap of LFWFs

$$\begin{aligned} q^2=0 \\ q^2=-q_1^2 \end{aligned} \quad M^2 = \sum_i \frac{k_{1i}^2 + m^2}{x_i} \rightarrow \sum_i \frac{(k_{1i}^2 + (\delta_{ii} - x_i) q_{1i}^2) + m^2}{x_i}$$

$$\Rightarrow F_i(q^2) \sim Q (m^2 \sim O(Q^2))$$

$$\sim \left(\frac{1}{Q^2}\right) \text{ mod } \log Q^2$$

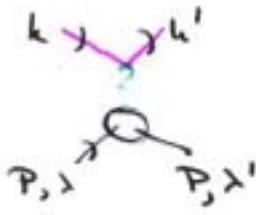
Agrees with PWD analysis

Belitsky
Ji
Yuan

$$\times \quad \frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim \log^2 Q^2$$

$$\text{not} \quad \frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim \text{const}$$

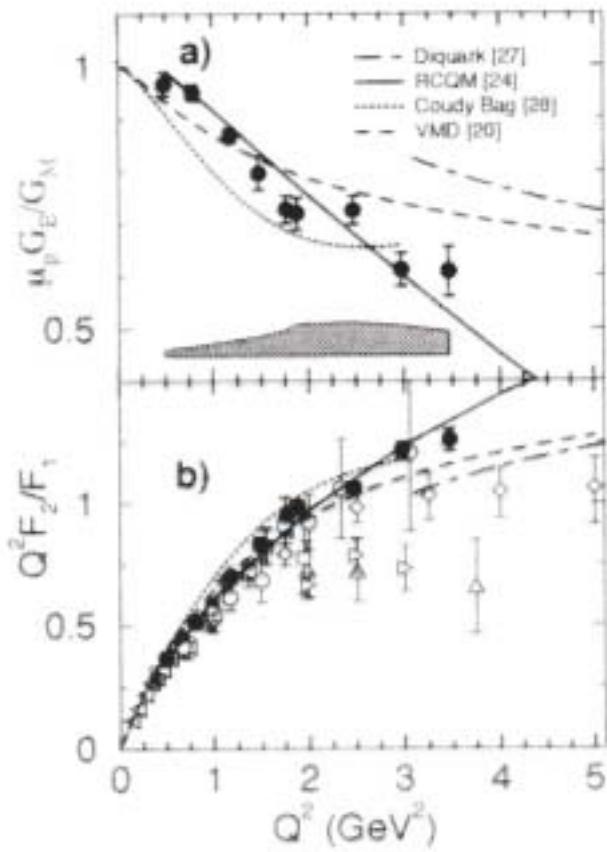
Miller Fresh
Coester
Salling,
Rosen
Kroll



JLab uses polarization transfer
to recoil proton

$$\ell^\mu \langle p', \lambda' | j_\mu | p, \lambda \rangle \propto G_E, G_M$$

$$\frac{p_x}{p_y} = \frac{G_E}{G_M} \frac{2M}{(kT)^2} e^{\frac{-2M}{kT}}$$



• Jefferson
Lab

PQCD

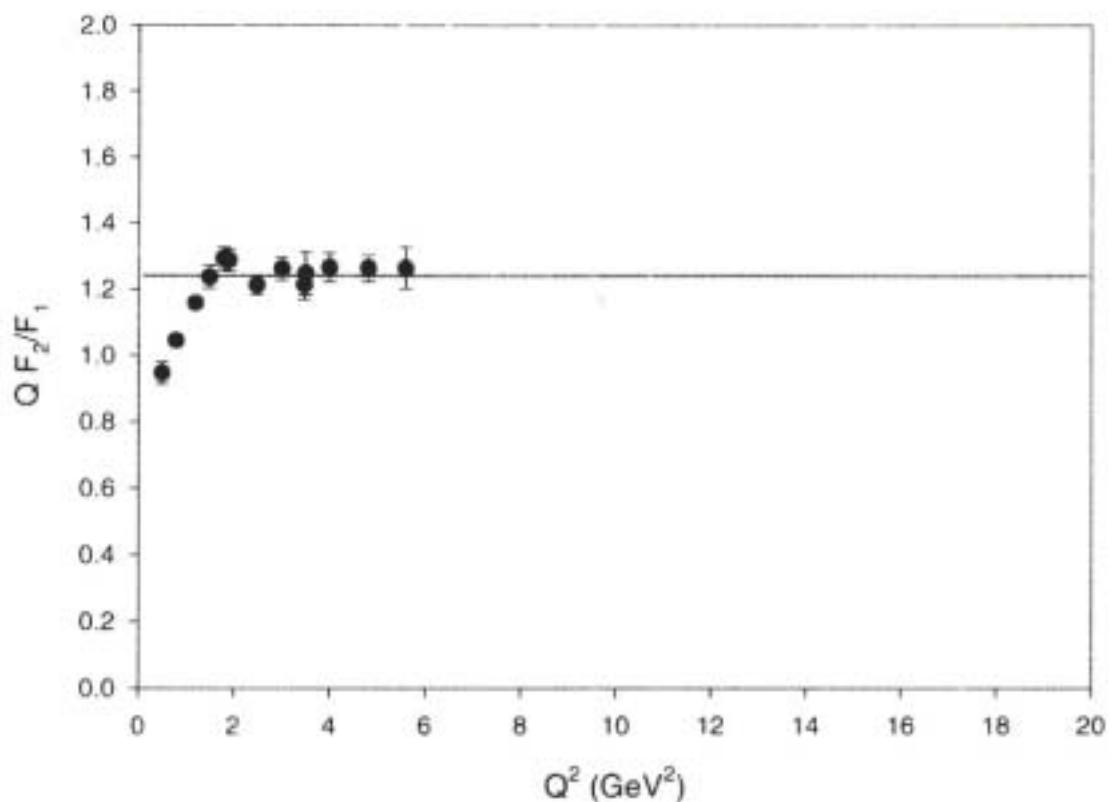
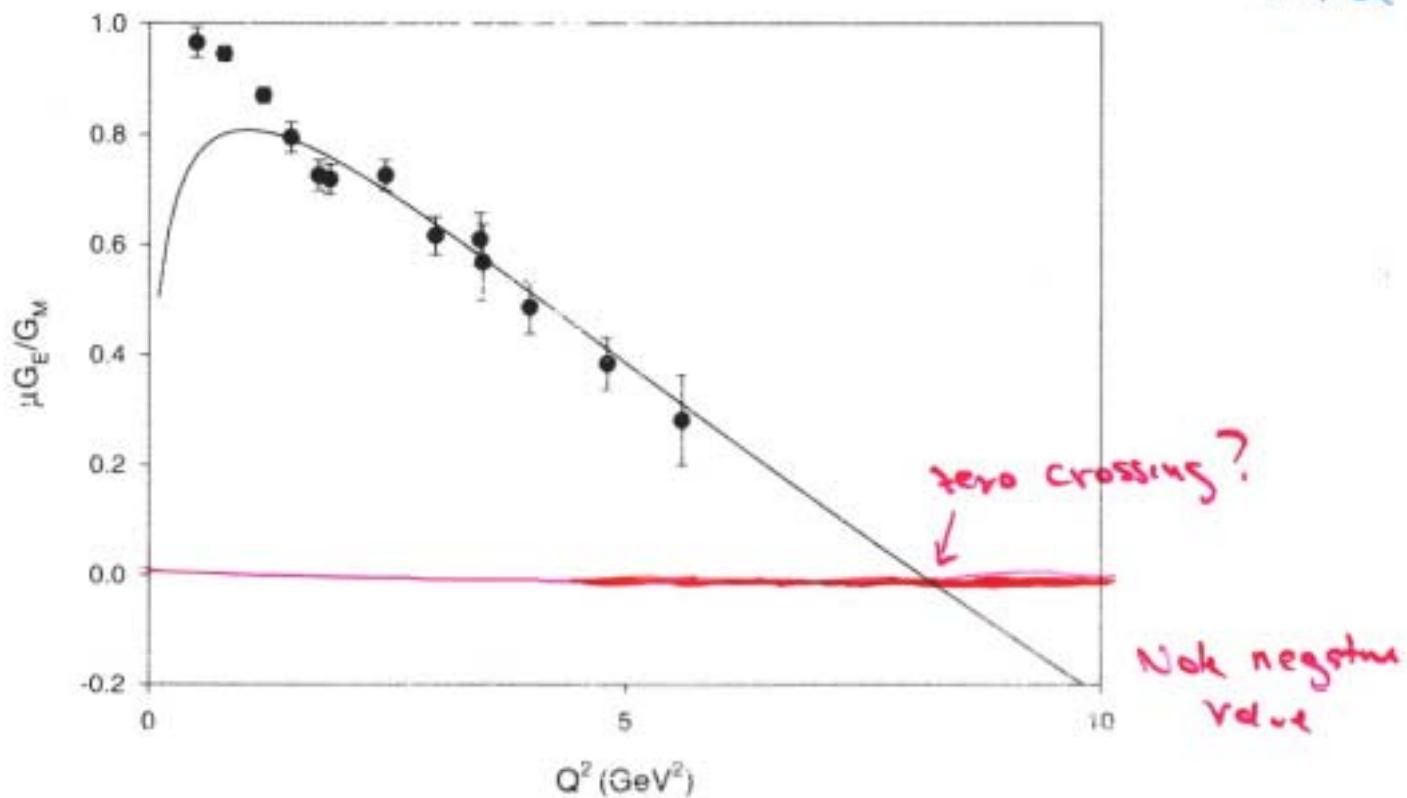
$$\frac{F_2}{F_1} \sim \frac{1}{Q^2}$$

diquark
model?

$$F_2/F_1 = 0.66/\sqrt{\tau}$$

$$\tau = Q^2/4\pi^2$$

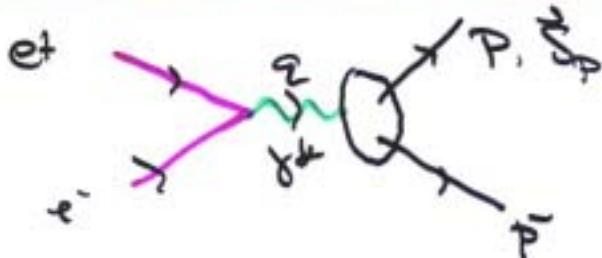
J.H. Linn



Timelike Determination of Relative Phase of Form Factors

↙ key measure to settle Rosenbluth
vs Spin Transfer

Single-Spin Polarization in $e^+e^- \rightarrow p + \bar{p}$



Carbon shell target
Use second Scott
to determine \tilde{S}_p

$$\tau = q^2/4m_e^2 > 1$$

$$P_y = \frac{\sqrt{\tau} \sin 2\theta \operatorname{Im} G_E^* G_M}{\tau |G_M|^2 (1 + \cos^2 \theta) + |G_E|^2 \sin^2 \theta}$$

Dubnicka
Dubnicka
Strutinski
Meloche

Rock

Carlson
Miller
Muay
SYB

$$\vec{S} \cdot (\vec{P}_p \times \vec{k}_e)$$

correction

normal to production
plane

$$\operatorname{Im} G_E^* G_M = (\epsilon - 1) \operatorname{Im} F_1 F_0$$

Various models for $G_E(q^2)$, $G_M(q^2)$
give very different relative phase.
when analytically continued for $q^2 < 0$
to $q^2 > 4m_p^2$.

$\leftarrow \text{regime } P_C \rightarrow$

$$P_x, P_y, P_z$$

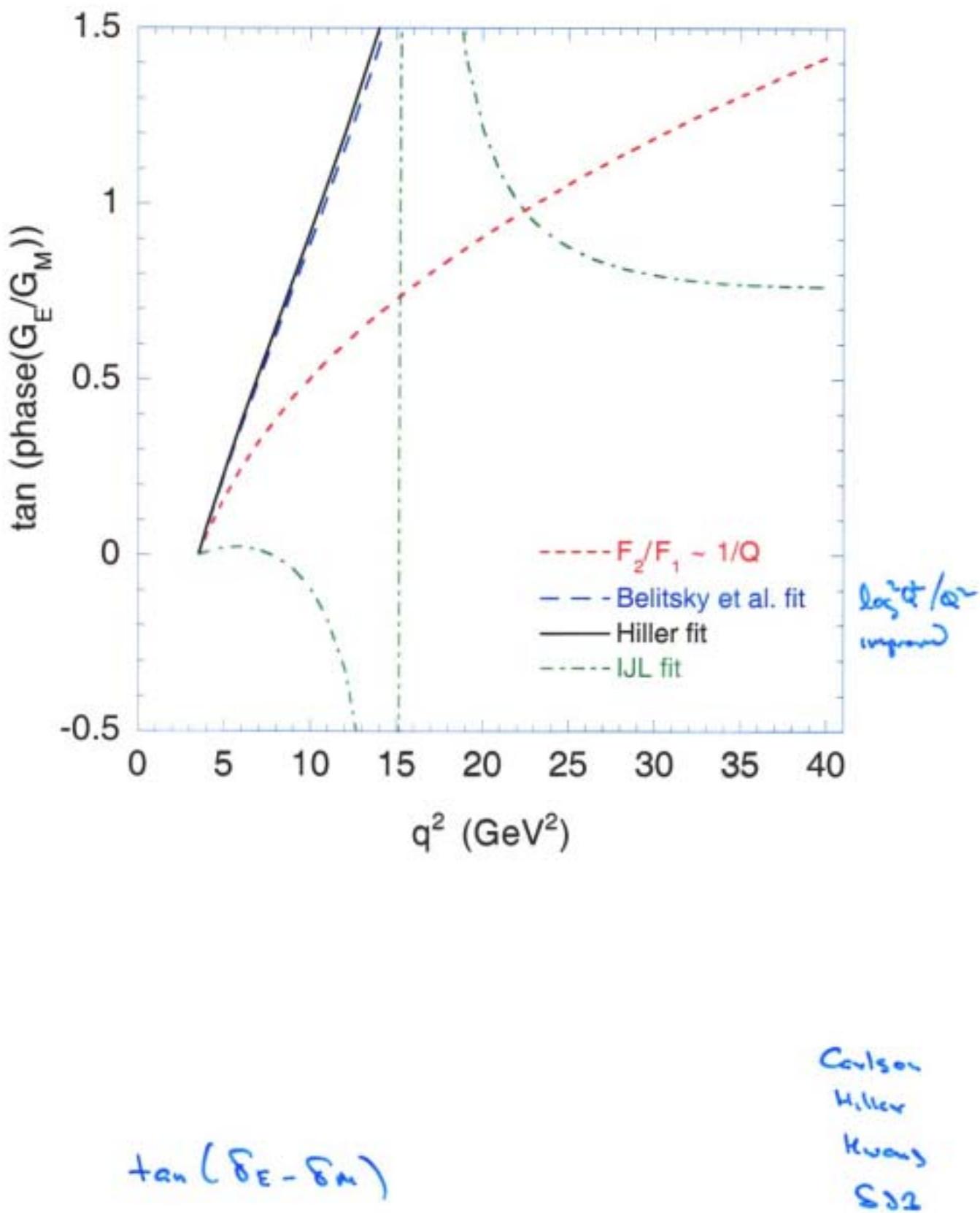
$$x \quad P_y = \sqrt{\tau} \frac{\sin 2\theta \operatorname{Im} G_E^* G_M}{D}$$

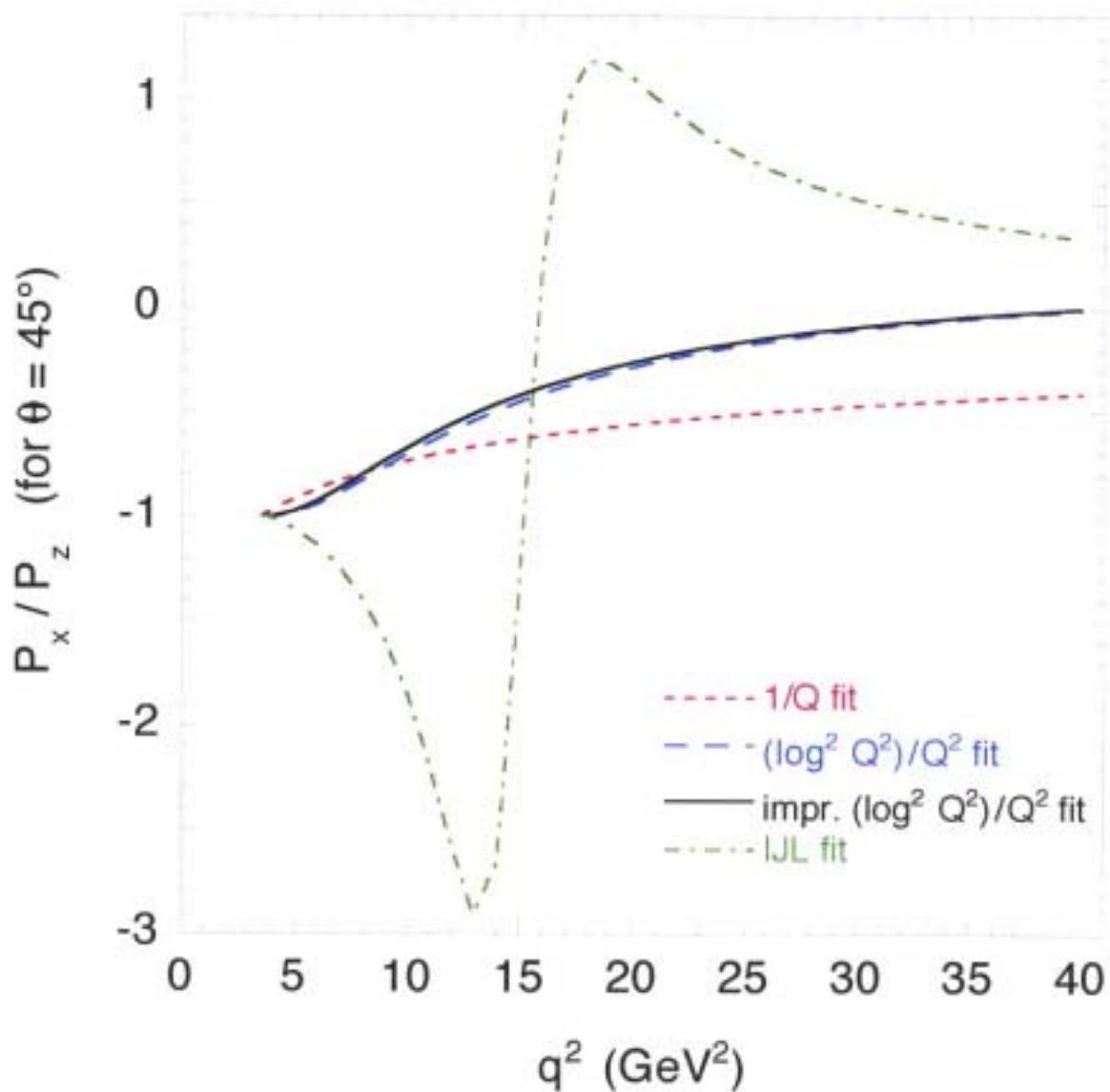
$$x \quad P_x = -P_e \frac{2\sqrt{\tau} \sin \theta \operatorname{Re} G_E^* G_M}{D}$$

$$x \quad P_z = -P_e \frac{2\tau \cos \theta |G_M|^2}{D}$$

$$D = \tau |G_M|^2 (1 + \cos^2 \theta) + |G_E|^2 \sin^2 \theta$$

$$\tau = q^2 / 4m_p^2 \gg$$

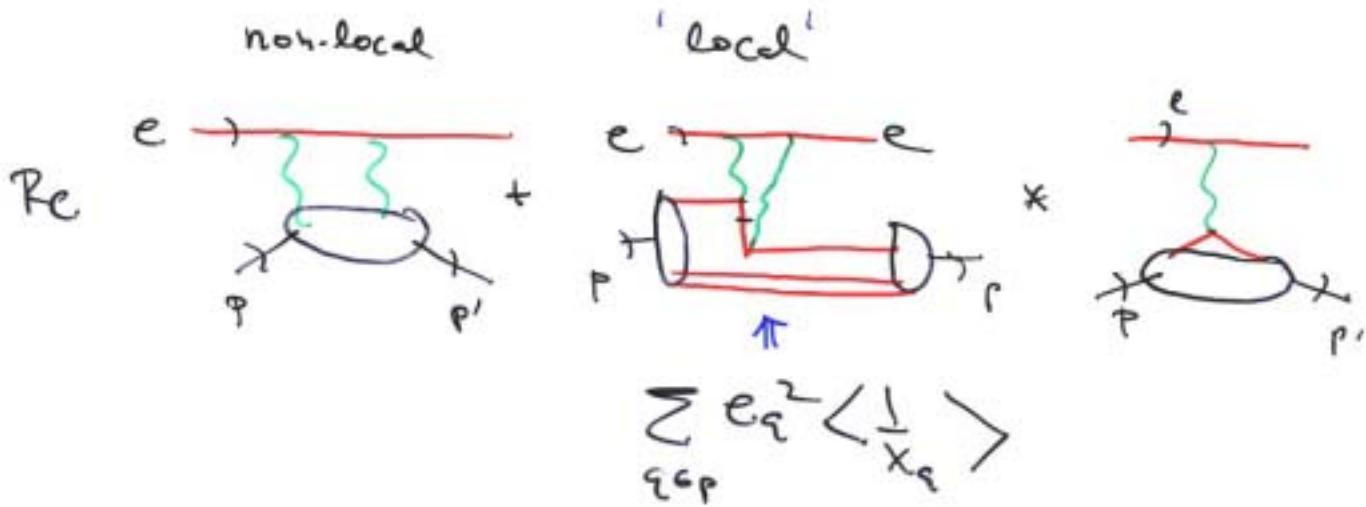




BC4n

Guido Vanenloogen
Blunden, Melville, et al.

$e^+ p \rightarrow e^+ p$ Asymmetry



" $J=0$ " Fixed Pole
 $M \sim S^0 F(+)$

π^2 enhancement in space-like scattering.

$$\delta M_p^2 = \delta m_p^2 \left\langle \frac{1}{x_q} \right\rangle$$

some moment.

Afanasov

Carlson

Test in front-back asymmetry in $e^- e^- \rightarrow H \bar{H}$

